Distributed Optimization in Directed Graphs: Push-Sum Based Algorithms
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We consider distributed optimization by a collection of nodes, each having access to its own convex function, whose collective goal is to minimize the sum of the functions. The communications between nodes are described by a time-varying sequence of directed graphs, which is uniformly strongly connected. For such communications, assuming that every node knows its outdegree, we develop a broadcast-based algorithm, termed the subgradient-push, which steers every node to an optimal value under a standard assumption of subgradient boundedness. The subgradient-push requires no knowledge of either the number of agents or the graph sequence to implement. Our analysis shows that the subgradient-push algorithm converges at a rate of $O(\ln t/t)$.

The proportionality constant in the convergence rate dependson the initial values at the nodes, the subgradient norms and, more interestingly, on both the speed of the network information diffusion and the imbalances of influence among the nodes.